Geometry: 9.4-9.7 Notes

NAME______________________

9.4 Use the Tangent Ratio______________ ______________________________ Date:________________

Define Vocabulary:

trigonometric ratio –

tangent –

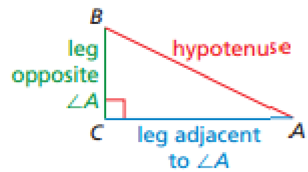
angle of elevation –

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as $\tan A$) is defined as follows.

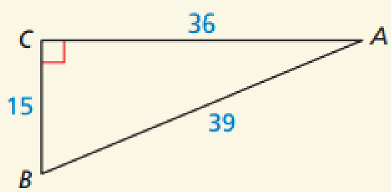
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



Examples: Finding Tangent Ratios

<u>WE DO</u>

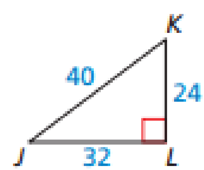
Find tan *A* and tan *B*. Write each answer as a fraction and as a decimal rounded to four places.



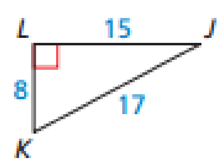
<u>YOU DO</u>

Find tan J and tan K. Write each answer as a fraction and as a decimal rounded to four places.

1.



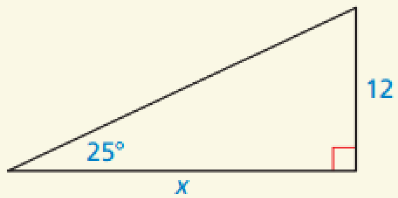
2.



Examples: Finding a leg length.

WE DO

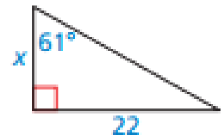
Find the value of *x*. Round your answer to the nearest tenth.



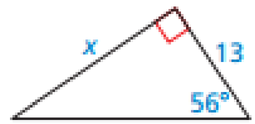
YOU DO

Find the value of x. Round your answer to the nearest tenth.

3.



4.



Examples: Using Special Right triangle to find a Tangent.

WE DO

Use a special right triangle to find the tangent of a 30° angle.

YOU DO

Use a special right triangle to find the tangent of 45°.

Examples: Modeling with Mathematics

WE DO

You are measuring the height of a tree. You stand 40 feet from the base of the tree. The angle of elevation to the top of the tree is 65°. Find the height of the tree to the nearest foot.

YOU DO

You are measuring the height of a lamppost. You stand 40 inches from the base of the lamppost. You measure the angle of elevation from the ground to the top of the lamppost to be 70°. Find the height h of the lamppost to the nearest inch.

Assignment	

9.5 Use Sine and Cosine Ratios________________ _________________________ Date:____________

Define Vocabulary:

sine –

cosine –

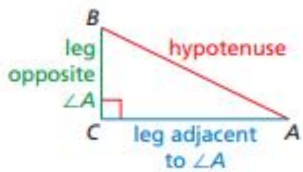
angle of depression –

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as $\sin A$ and $\cos A$) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

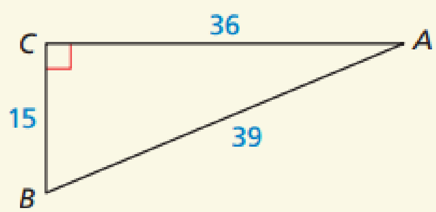
$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



Examples: Finding Sine and Cosine Ratios.

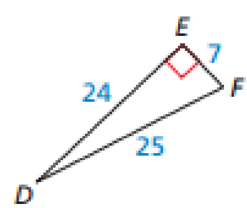
<u>WE DO</u>

Find sin *A*, sin *B*, cos *A*, and cos *B*. Write each answer as a fraction and as a decimal rounded to four places.



<u>YOU DO</u>

Find sin D, sin F, cos D, and cos F. Write each answer as a fraction and as a decimal rounded to four places.



Sine and Cosine of Complementary Angles

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

Let A and B be complementary angles. Then the following statements are true.

$\sin A = \cos(90° - A) = \cos B$ $\quad$ $\sin B = \cos(90° - B) = \cos A$

$\cos A = \sin(90° - A) = \sin B$ $\quad$ $\cos B = \sin(90° - B) = \sin A$

Examples: Rewriting Trigonometric Expressions.

WE DO

Write cos 68° in terms of sine.

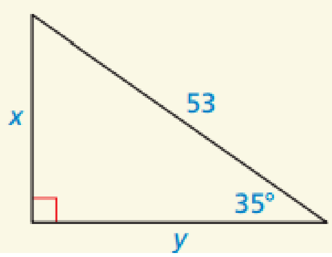
YOU DO

Write cos 23° in terms of sine.

Examples: Finding Leg Lengths

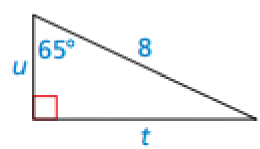
WE DO

Find the values of x and y using sine and cosine. Round your answers to the nearest tenth.



YOU DO

Find the values of u and t using sine and cosine. Round your answers to the nearest tenth.

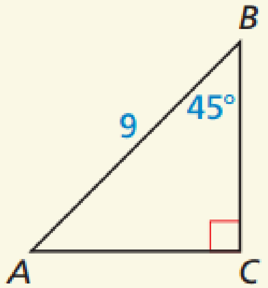


Examples: Finding Sine and Cosine in Special Right Triangles.

<u>WE DO</u>

Which ratios are equal to $\frac{\sqrt{2}}{2}$? Select all that apply.

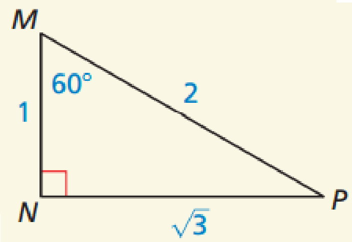
- $\sin A$
- $\cos A$
- $\tan A$
- $\sin B$
- $\cos B$
- $\tan B$



<u>YOU DO</u>

Which ratios are equal to $\frac{\sqrt{3}}{2}$? Select all that apply.

- $\sin M$
- $\sin P$
- $\cos M$
- $\cos P$



Examples: Modeling with Mathematics

<u>WE DO</u>

You are skiing down a hill with an altitude of 800 feet. The angle of depression is 15°. Find the distance x you ski down the hill to the nearest foot.

<u>YOU DO</u>

A camera attached to a kite is filming the damage caused by a brush fire in a closed-off area. The camera is directly above the center of the closed-off area. A person is standing 100 feet away from the center of the closed-off area. The angle of depression from the camera to the person flying the kite is   How long is the string on the kite?

Assignment	

Define Vocabulary:

inverse tangent –

inverse sine –

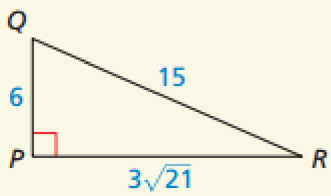
inverse cosine –

solve a right triangle –

Examples: Identifying Angles from Trigonometric Ratios

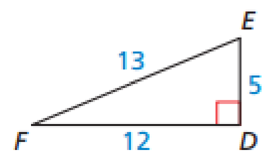
<u>WE DO</u>

Determine which of the two acute angles has a sine of 0.4.



<u>YOU DO</u>

Determine which of the two acute angles has the given trigonometric ratio.

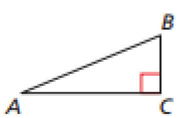


1. The sine of the angle is $\frac{12}{13}$.

2. The tangent of the angle is $\frac{5}{12}$.

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$

Examples: Finding Angle Measures

WE DO

Let ∠A, ∠B, and ∠C be acute angles. Use a calculator to approximate the measures of ∠A, ∠B, and ∠C to the nearest tenth of a degree.

a. tan A = 3.29

b. sin B = 0.55

c. cos C = 0.87

YOU DO

Let ∠G, ∠H, and ∠K be acute angles. Use a calculator to approximate the measures of ∠G, ∠H, and ∠K to the nearest tenth of a degree.

a. tan G = 0.43

b. sin H = 0.68

c. cos K = 0.94

Solving a Right Triangle

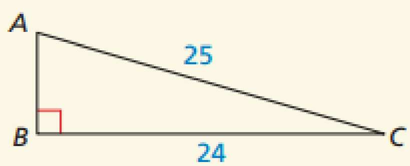
To **solve a right triangle** means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following.

- two side lengths
- one side length and the measure of one acute angle

Examples: Solving a Right Triangle

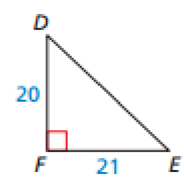
WE DO

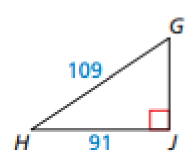
Solve the right triangle. Round decimal answers to the nearest tenth.



YOU DO

Solve the right triangle. Round decimal answers to the nearest tenth.

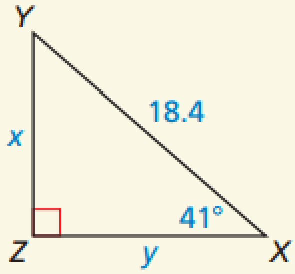




Examples: Solving a Right Triangle

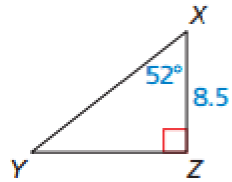
WE DO

Solve the right triangle. Round decimal answers to the nearest tenth.



YOU DO

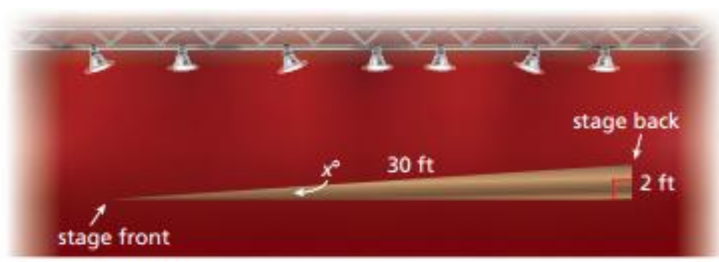
Solve the right triangle. Round decimal answers to the nearest tenth.



Examples: Solving a Real-Life Problem

WE DO

Use the information in Example 5. Another raked stage is 25 feet long from front to back with a total rise of 1.5 feet. You want the rake to be 5° or less. Is the raked stage within your desired range? Explain.



Assignment	

9.7 Use Sine and Cosine Ratios__________________ ____________________________ Date:_____________

Define Vocabulary:

Law of Sines

Law of Cosines

Examples: Finding Trigonometric Ratios for Obtuse Angles

WE DO

Use a calculator to find each trigonometric ratio. Round your answer to four decimal places.

a. tan 92°

b. sin 175°

c. cos 149°

YOU DO

Use a calculator to find each trigonometric ratio. Round your answer to four decimal places.

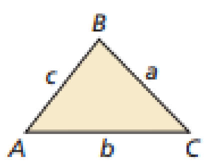
a. tan 110°

b. sin 97°

c. cos 165°

Area of a Triangle

The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area.

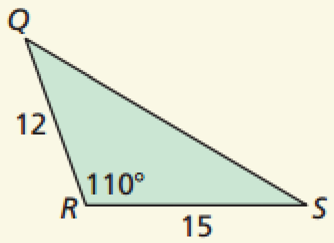


Area $= \frac{1}{2}bc \sin A$ Area $= \frac{1}{2}ac \sin B$ Area $= \frac{1}{2}ab \sin C$

Examples: Finding the Area of a Triangle

<u>WE DO</u>

Find the area of the triangle. Round your answer to the nearest tenth.



<u>YOU DO</u>

Find the area of △ABC with the given side lengths and included angle. Round your answer to the nearest tenth.

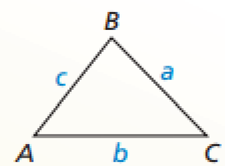
a. m∠B = 60°, a = 19, c = 14

b. m∠C = 29°, a = 38, b = 31

Theorem 9.9 Law of Sines

The Law of Sines can be written in either of the following forms for △*ABC* with sides of length *a*, *b*, and *c*.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

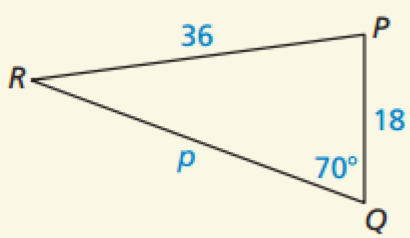


Proof Ex. 51, p. 516

Examples: Using the Law of Sines (SSA Case)

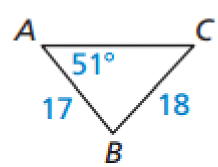
<u>WE DO</u>

Solve the triangle. Round decimal answers to the nearest tenth.



<u>YOU DO</u>

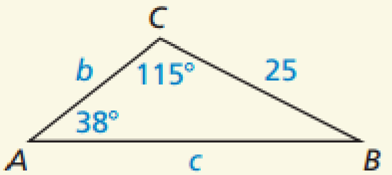
Solve the triangle. Round decimal answers to the nearest tenth.



Examples: Using the Law of Sines (AAS Case)

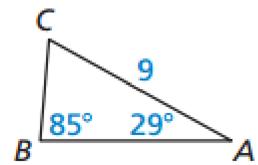
WE DO

Solve the triangle. Round decimal answers to the nearest tenth.



YOU DO

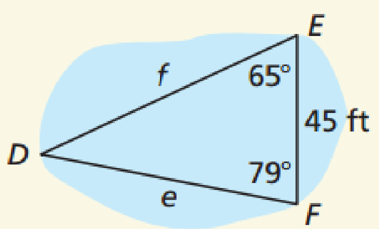
Solve the triangle. Round decimal answers to the nearest tenth.



Examples: Using the Law of Sines (ASA Case)

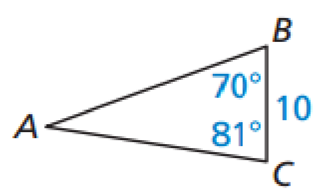
WE DO

A surveyor makes the measurements shown to determine the length *f* of a walking bridge to be built across a pond in a city park. Find the length of the bridge $\overline{DE}$ to the nearest tenth.



YOU DO

Solve the triangle. Round decimal answers to the nearest tenth.



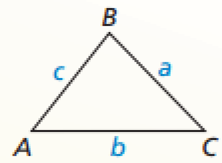
Theorem 9.10 Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, as shown, then the following are true.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

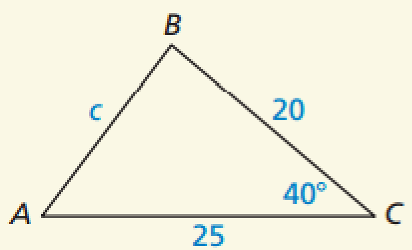


Proof Ex. 52, p. 516

Examples: Using the Law of Cosines (SAS Case)

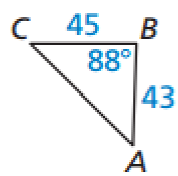
<u>WE DO</u>

Solve the triangle. Round decimal answers to the nearest tenth.



<u>YOU DO</u>

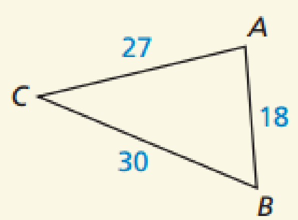
Solve the triangle. Round decimal answers to the nearest tenth.



Examples: Using the Law of Cosines (SSS Case)

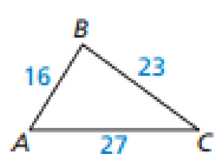
<u>WE DO</u>

Solve the triangle. Round decimal answers to the nearest tenth.



<u>YOU DO</u>

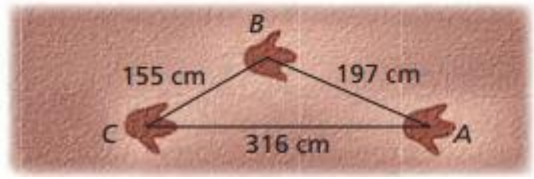
Solve the triangle. Round decimal answers to the nearest tenth.



Examples: Solving a Real-Life Problem

WE DO

Use the information in Example 8. Another dinosaur's footprints showed that the distance from C to B is 1.5 meters, the distance from B to A is 2 meters, and the distance from C to A is 3.2 meters. Find the step angle B to the nearest tenth.



Assignment	